Dynamical correlation length near the chiral critical point

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Abstract. The dynamical evolution of small systems undergoing a chiral symmetry breaking transition in the course of rapid expansion is discussed. The time evolution of the dynamical correlation length for trajectories passing through a second-order critical point is extracted. It is shown that while the maximum value of the correlation length is bound from above by dynamical effects, the time interval during which it is near its maximum grows steadily with the system size and with decreasing expansion rate.

1 Introduction

The main goal of colliding heavy ions at high energies is to produce matter at high temperature and baryon density. In matter at such extreme conditions chiral symmetry may be approximately restored (for a review of signatures for hot and dense matter see e.g. [1]). In particular, it has been argued that a line of first-order phase transitions in the phase diagram of QCD could end in a second-order critical point, where the correlation length diverges [2]. The critical point in the temperature vs. baryon-chemical potential plane has been located on the lattice [3]. Here, we study the dynamics of the chiral fields near a critical point and determine the behavior of the correlation length in finite and rapidly expanding systems, such as the ones encountered in heavy-ion collisions.

2 Chiral hydrodynamics

In Chiral Hydrodynamics [4,5] it is assumed that the long-wavelength (classical) modes of the chiral fields evolve in the effective potential generated by the thermalized degrees of freedom, which are the matter fields (and possibly also hard modes of the chiral field). In our model, the latter are described as a perfect relativistic fluid, whose equation of state is in turn determined by the chiral field (via the effective mass), and which can exchange energy and momentum with the chiral fields. The chiral symmetry breaking dynamics is described by an effective field theory, in our case the SU(2)×SU(2) linear σ-model:

$$\mathcal{L} = \overline{q} \left[i\gamma^\mu \partial_\mu - g(\sigma + \gamma_5 \tau \cdot \pi) \right] q$$
$$+ \frac{1}{2} \left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi \right) - U(\sigma, \pi) \quad . \quad (1)$$

The potential $U(\sigma, \pi)$, which exhibits both spontaneously and explicitly broken symmetry, is given by

$$U(\sigma, \pi) = \frac{\lambda^2}{4} (\sigma^2 + \pi^2 - v^2)^2 - h_q \sigma \quad . \quad (2)$$

Here q is the constituent-quark field $q = (u, d)$. The scalar field σ and the pseudoscalar field π together form a chiral field $\phi = (\sigma, \pi)$. The vacuum expectation values of the condensates are $\langle \sigma \rangle = f_\pi$ and $\langle \pi \rangle = 0$, where $f_\pi = 93$ MeV is the pion decay constant.

For $g > 0$, the finite-temperature one-loop effective potential includes a contribution from the quarks, and is given by

$$V_{\text{eff}}(\phi, T) = U(\phi) - d_q \int \frac{d^3\mathbf{p}}{(2\pi)^3} T \log \left[1 + e^{-E/T} \right] \quad (3)$$

V_{eff} depends on the order parameter field through the effective mass of the quarks, $m_q = g|\phi|$, which enters the expression for the energy E.

For sufficiently small g one finds a smooth transition between the two phases. For large coupling, however, the effective potential exhibits a first-order phase transition [6]. Along the line of first-order transitions the effective potential has two minima. At $T = T_c$ these two minima are degenerate and are seperated by a barrier. As one lowers the value of g the barrier gets smaller and the two minima approach each other. At $g_c \simeq 3.7$, finally, the barrier vanishes, and so does the latent heat. Below, we study the hydrodynamic expansion near this chiral critical point.

The classical equations of motion for the chiral fields are

$$\partial_\mu \partial^\mu \phi + \frac{\delta V_{\text{eff}}}{\delta \phi} = 0 \quad (4)$$

The dynamical evolution of the thermalized degrees of freedom (fluid of quarks) is determined by the local conser-

vation laws for energy and momentum. Note that we do not assume that the chiral fields are in equilibrium with the heat bath of quarks. Hence, the fluid pressure p depends explicitly on $|\phi|$, see [5] for more details. Due to the interaction between fluid and field the *total* energy and momentum is the conserved quantity:

$$\partial_\mu \left(T^{\mu\nu}_{\text{fluid}} + T^{\mu\nu}_\phi \right) = 0. \tag{5}$$

We emphasize that we employ (4) not only to propagate the mean field through the transition but fluctuations as well. The initial condition includes some generic "primordial" spectrum of fluctuations (see below) which then evolve in the effective potential generated by the matter fields. Near the critical point, those fluctuations have small effective mass and "spread out" to probe the flat effective potential.

3 Correlation Length

The correlation length provides a measure for the typical length scale over which fluctuations of the fields are correlated. Following the Ginzburg-Landau theory, for an infinite system in global thermal equilibrium one can expand the free energy F arround the thermodynamic expectation value ϕ_0 – which is given by the minimum of the free energy – in a power series:

$$F(\bar\phi) = a_0 + a_1\bar\phi + a_2\bar\phi^2 + a_3\bar\phi^3 + a_4\bar\phi^4 + \ldots + a_{\text{N}}\bar\phi^N \tag{6}$$

where $\bar\phi = \frac{\phi - \phi_0}{f_\pi}$ and $a_{\text{N}} > 0$. The correlation length ξ is defined as the second derivative of the free energy

$$\frac{1}{\xi^2} = \left. \frac{d^2 F(\bar\phi)}{d\bar\phi^2} \right|_{\bar\phi=0} = 2a_2. \tag{7}$$

The correlation length is finite for $T \neq T_c$ since the effective potential has a finite curvature there. At T_c the potential about ϕ_0 becomes flat and thus the correlation length diverges as the system approaches the critical point. Of course, this can only be true for an infinite system in global thermodynamic equilibrium. In high-energy heavy ion collisions instead one deals with finite and rapidly expanding systems where the true dynamical correlation length is finite.

4 Results

4.1 Initial conditions

To study the effects of a finite and expanding system we choose the following initial conditions. The distribution of the fluid energy density at $t = 0$ is taken to be spherical:

$$e(t = 0, r) = e_{\text{eq}}\, \Theta(R - r) , \tag{8}$$

where e_{eq} denotes the equilibrium value of the energy density taken at a temperature of $T_i \approx 160$ MeV which is well

above the transition temperature $T_c \approx 138$ MeV. R denotes the initial size of the system. For the fluid velocity we assume a linear profile

$$v_r(t = 0, r) = \frac{r}{R}\, \Theta(R - r) . \tag{9}$$

Note that the expansion rate (Hubble constant) equals $1/R$, hence larger systems also correspond to less rapid expansion.

The initial conditions for the chiral fields are

$$\sigma(t = 0, \mathbf{x}) = \delta\sigma(\mathbf{x}) + f_\pi +$$
$$(-f_\pi + \sigma_{\text{eq}}) \cdot \left[1 + \exp\left(\frac{r - R}{a} \right) \right]^{-1}$$
$$\pi(t = 0, \mathbf{x}) = \delta\pi(\mathbf{x}) , \tag{10}$$

$a = 0.3$ fm is the surface thickness of this Woods-Saxon like distribution. Here $\sigma_{\text{eq}} \approx 0$ is the value of the σ field corresponding to e_{eq}. Thus, the chiral condensate nearly vanishes at the center, where the energy density of the quarks is large, and then quickly interpolates to f_π where the matter density is low.

$\delta\sigma(\mathbf{x})$ and $\delta\pi(\mathbf{x})$ represent Gaussian random fluctuations of the fields. We correlate them over an initial correlation length of $\xi_0 \approx 1.2$ fm as described in [5]. Our focus now is on how those "primordial" fluctuations evolve through the transition and how the correlation length depends on the system size.

4.2 Extraction of correlation length

To analyse the time evolution of ξ in our finite and expanding system we need to know the *dynamical* effective potential probed by the fluctuating fields. This is done by extracting a histogram of the field distribution at every time step, within a sphere of radius 1 fm around $r = 0$. This histogram has been averaged over a few random initial field configurations, picked according to (10).

The probability distribution is related to the 4-d effective action by

$$P[\phi] \propto \exp\{-S_{\text{eff}}\} , \tag{11}$$

and thus we can extract the effective action "seen" by the chiral fields by fitting a polynomial of the form (6) to

$$S_{\text{eff}}(\phi) \propto -\log\{P[\phi]\} . \tag{12}$$

4.3 Numerical results

Figure 1 depicts the time evolution of the extracted correlation length ξ for different system sizes. Evidently, the correlation length is always *finite*. For a system of initial radius $R = 1$ fm, $\xi(t) \approx \xi_0$ remains approximately constant during the expansion, equal to the initial correlation length. However, for larger R, it develops a maximum at intermediate times, roughly twice ξ_0. Its maximum value

appears to grow only very slowly with R (once $R \geq 3$ fm), i.e. the approach towards $\xi = \infty$ for $R \to \infty$ is slow. Both observations are in line with [7] who studied finite-time effects for infinitely large systems, and estimated that ξ can not exceed $2-3$ times ξ_0, and that the maximum value of ξ grows only slowly with decreasing cooling rate. This implies that experimental observables will not exhibit "fully developped" critical behavior even for trajectories through the critical point. On the other hand, one might hope for some signals to show up even for trajectories that miss the critical point somewhat, since the growth of ξ is damped by dynamical effects anyways. We also observe from our real-time analysis that the time interval during which ξ is near its maximum *grows* steadily with R. This might be relevant for observables that integrate over the entire collision history.

The behavior of other couplings a_i $(i > 2)$ belonging to marginal or irrelevant operators will be reported elsewhere.

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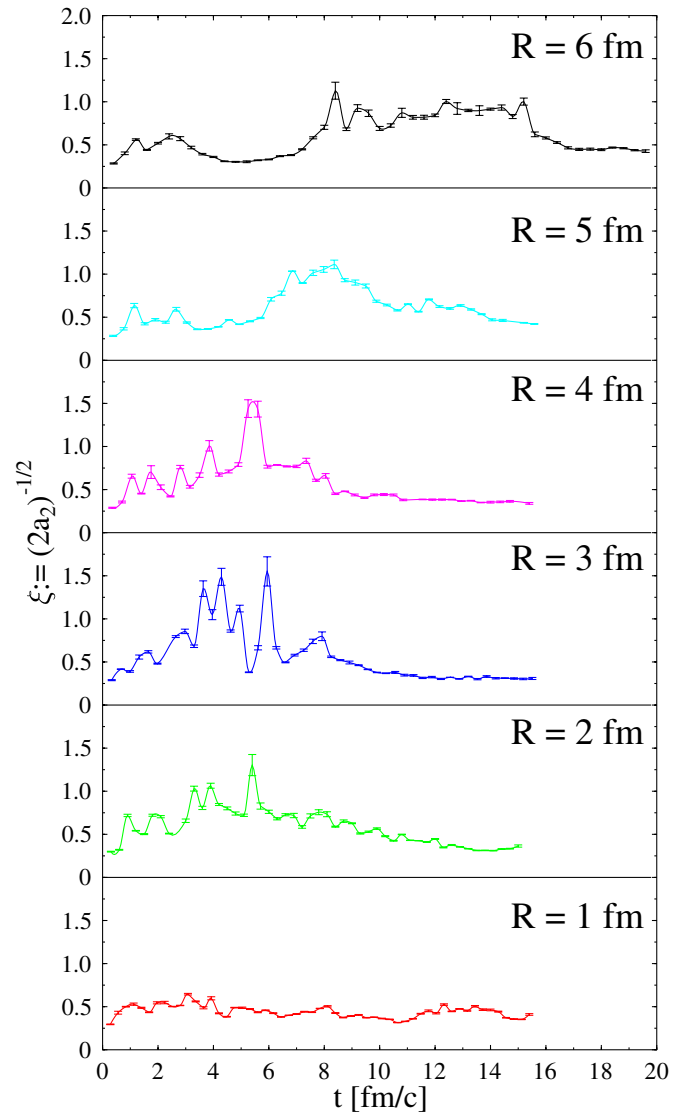7. B. Berdnikov, K. Rajagopal: Phys. Rev. D **61** (2000) 105017

Fig. 1. Time evolution of the correlation length ξ for finite and expanding systems of different initial radius R.